

Comparison of canonical and grand canonical models for selected multifragmentation data

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Calculations for a set of nuclear multifragmentation data are made using a canonical and a grand canonical model. The physics assumptions are identical but the canonical model has an exact number of particles, whereas the grand canonical model has a varying number of particles, hence, is less exact. Interesting differences are found.

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I. INTRODUCTION

In experiments whose goals were to investigate the role of isospin in fragment yields [1], the following interesting features have been observed. If we compare the central collisions of two heavy ion systems 1 and 2, which are similar in all aspects of the reactions except for the neutron and proton composition, the isotope yield ratios, $Y_2(n,z)/Y_1(n,z)$, where $Y_i(n,z)$ is the yield of the isotope with neutron number n , and proton number z , from reaction i , is found to exhibit an exponential relationship as a function of n and z [1,2],

$$Y_2(n,z)/Y_1(n,z) = C \exp(\alpha_n n + \alpha_p z), \quad (1.1)$$

where C is an overall normalization constant and α_n and α_p are fitting parameters. This phenomenon is termed isoscaling, a strong evidence that the processes are statistical.

Related to isoscaling is the exponential dependence of the mirror-nuclei ratios on the binding energy. In Fig. 1, the ratios of yields of mirror nuclei: $Y_i(t)/Y_i(^3\text{He})$, $Y_i(^7\text{Li})/Y_i(^7\text{Be})$ and $Y_i(^{11}\text{B})/Y_i(^{11}\text{C})$ for central collisions of $^{124}\text{Sn}+^{124}\text{Sn}$ (solid points) and $^{112}\text{Sn}+^{112}\text{Sn}$ (open points) at 50 MeV per nucleon are plotted as a function of the binding energy difference ΔE_B . These ratios fall approximately on an exponential. Many statistical models such as the grand canonical model [3,4] of multifragmentation predict both the isoscaling and mirror-nuclei ratio dependence.

Experimental evidence suggests that multifragmentation occurs when the heated matter expands to density about one-third of nuclear matter density [5] and the time scale for the emission of fragments is short, between 50 and 100 fm/c [6]. Most successful statistical models that describe multifragmentation data assume a freeze-out volume at which composite yields are to be calculated entirely according to phase space [7,8]. If the dissociating system is very large, then grand canonical simplification can be employed [3,4]. According to this model, the average number of composites with neutron number i and proton number j is

$$\langle n_{i,j} \rangle = \exp[\beta(i\mu_n + j\mu_p)] \omega_{i,j}, \quad (1.2)$$

where μ_n, μ_p are neutron and proton chemical potentials and

$$\omega_{i,j} = \frac{V}{h^3} [2\pi(i+j)mT]^{3/2} (2s+1) z_{int} \exp(\beta E_B) \quad (1.3)$$

is the partition function of one composite. β is the inverse temperature. For mirror nuclei: $i = k+1, k$ and $j = k, k+1$ we should simply have

$$\frac{\langle n_{k+1,k} \rangle}{\langle n_{k,k+1} \rangle} = \exp(\beta\mu_n - \beta\mu_p) \exp(\beta\Delta E_B). \quad (1.4)$$

Thus the log of the ratios of the yield will be linear with respect to ΔE_B that is approximately obeyed by data. However a more close inspection raises another issue.

According to Eq. (1.4), one can deduce the value of $\beta = 1/T$ from the slope of the line. Indeed for the lines drawn in Fig. 1, the temperature T is less than 2 MeV. For such a low temperature, the model of simultaneous breakup model [7,8] should not be appropriate. In addition, such low values are in direct contradictions with temperature measurements obtained from isotope yield ratios. The isotope yield temperature is about 5 MeV for the Sn+Sn systems [9,10]. To

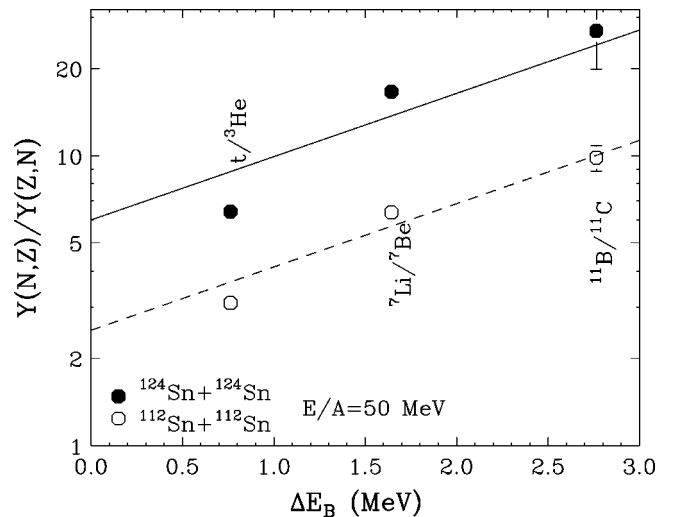


FIG. 1. Isobar ratios for three pairs of mirror nuclei obtained from the central collisions of $^{124}\text{Sn}+^{124}\text{Sn}$ (solid points) and $^{112}\text{Sn}+^{112}\text{Sn}$ (open points) collisions. The lines are best fit of Eq. (1.4).

resolve the discrepancies between temperatures observed, it is necessary to explore details of the exponential behavior of the mirror nuclei.

II. CANONICAL VS GRAND CANONICAL MODELS

In recent years, the grand canonical model has been replaced by a canonical model. The physics assumptions are still the same but we no longer have to assume that the system is large. This is a technical advancement; the details have already been described in several places [11–13] so we will not repeat these here. The model has been used to fit the isotope data [12,14]. Surprisingly, isoscaling that follows naturally from the grand canonical model, emerges also in canonical model [14]. In this paper, we will investigate why certain results from the canonical model resemble those from the grand canonical model and what are the differences. We will also investigate the relation between the canonical temperature and the temperature obtained based on the simpler grand canonical rules.

The yield of the composite that has $k+1$ neutrons and k protons is given in the canonical model by

$$\langle n_{k+1,k} \rangle = \omega_{k+1,k} \frac{Q_{N-k-1,Z-k}}{Q_{N,Z}}. \quad (2.1)$$

Here N, Z refer to the number of neutrons and protons of the disintegrating system. $Q_{N,Z}$ is the canonical partition function of this system. Similarly, $Q_{N-k-1,Z-k}$ is the canonical partition function of the residue system that has $N-k-1$ neutrons and $Z-k$ protons. The ratio of the yields in the canonical model is then given by

$$\frac{\langle n_{k+1,k} \rangle}{\langle n_{k,k+1} \rangle} = \frac{\omega_{k+1,k}}{\omega_{k,k+1}} \frac{Q_{N-k-1,Z-k}}{Q_{N-k,Z-k-1}}. \quad (2.2)$$

The first factor leads to $\exp(\beta \Delta E_B)$. We note in passing that for mirror nuclei $\Delta E_B = \Delta E_C$, the change in Coulomb energy. If we assume a uniformly charged sphere, then $\Delta E_C = \frac{3}{5} e^2 / R_0 a^{1/3} [(z+1)^2 - z^2] = 0.72 a^{2/3}$ MeV, where a is the composite mass number. For light nuclei $0.72 a^{2/3}$ MeV does not fit the data very well. We note for later use that $0.235 a$ MeV fits ΔE_B between $a=7$ and $a=15$ better.

The exact expression for the canonical partition function $Q_{N,Z}$ used in [14] does not allow us to investigate easily the features we want to study. Since the ratios are very simple in the grand canonical ensemble and since there is a connection

between grand canonical partition function $Z_{gr}(\lambda_n, \lambda_p)$ [$\lambda = \beta \mu$, of Eq. (1.4)] and the canonical partition function $Q_{N,Z}$, we find it convenient to exploit this relation. In the present problem, the grand canonical partition function is given by

$$Z_{gr}(\lambda_n, \lambda_p) = \prod_{k,l} \sum_{n_{kl}} \exp[(k\lambda_n + l\lambda_p)n_{kl}] \frac{\omega_{k,l}^{n_{kl}}}{n_{kl}!}. \quad (2.3)$$

The expression for $\ln Z_{gr}(\lambda_n, \lambda_p)$ is

$$\ln Z_{gr}(\lambda_n, \lambda_p) = \sum_{k,l} \exp(k\lambda_n + l\lambda_p) \omega_{k,l}. \quad (2.4)$$

The canonical partition function can be obtained from Z_{gr} by Laplace inverse:

$$Q_{N,Z} = \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{-(\lambda_n + i\tilde{\lambda}_n)N} e^{-(\lambda_p + i\tilde{\lambda}_p)Z} \times \exp[\ln Z_{gr}(\lambda_n + i\tilde{\lambda}_n, \lambda_p + i\tilde{\lambda}_p)] d\tilde{\lambda}_n d\tilde{\lambda}_p. \quad (2.5)$$

While this expression is true for any λ_n and λ_p , the saddle-point approximation consists in choosing the values of λ_n and λ_p such that the kernel maximizes at $\tilde{\lambda}_n = 0$ and $\tilde{\lambda}_p = 0$ and making a Gaussian approximation for the integrand around this maximum. The result is

$$Q_{N,Z} \approx e^{-(\lambda_n N + \lambda_p Z)} \exp[\ln Z_{gr}(\lambda_n, \lambda_p)] / (2\pi^* |\det|^{1/2}), \quad (2.6)$$

where the values of λ_n and λ_p are such that the average numbers of neutrons and protons as obtained from the grand canonical ensemble are N and Z , i.e.,

$$N = \frac{\partial \ln Z_{gr}(\lambda_n, \lambda_p)}{\partial \lambda_n}, \quad (2.7)$$

$$Z = \frac{\partial \ln Z_{gr}(\lambda_n, \lambda_p)}{\partial \lambda_p}. \quad (2.8)$$

The elements of the determinant are given by $a_{1,1} = \partial^2 \ln Z_{gr} / \partial^2 \lambda_n$, $a_{1,2} = a_{2,1} = \partial^2 \ln Z_{gr} / \partial \lambda_n \partial \lambda_p$, and $a_{2,2} = \partial^2 \ln Z_{gr} / \partial^2 \lambda_p$.

Equation (2.2) now takes the form

$$\frac{\langle n_{k+1,k} \rangle}{\langle n_{k,k+1} \rangle} \approx e^{\beta \Delta E_B} \frac{\exp\{-[\lambda_n(N-k-1) + \lambda_p(Z-k)] + \ln Z_{gr}(\lambda_n, \lambda_p)\}}{\exp\{-[\lambda'_n(N-k) + \lambda'_p(Z-k-1)] + \ln Z_{gr}(\lambda'_n, \lambda'_p)\}}. \quad (2.9)$$

Here we have omitted the ratios of the determinants $|\det|^{1/2}$ because their effects will be negligible. Equation (2.9) will reduce to the standard grand canonical result if we set $\lambda_n = \lambda'_n$; $\lambda_p = \lambda'_p$ and take these values from a system that has

the average number of neutrons to be N (rather than $N-k-1$ to get λ_n and $N-k$ to get λ'_n) and the average number of protons to be Z (rather than $Z-k$ to obtain λ_p and $Z-k-1$ to obtain λ'_p).

For a better estimate, let us write $\lambda'_n = \lambda_n + \Delta\lambda_n$; $\lambda'_p = \lambda_p + \Delta\lambda_p$. Assuming lowest order expansion is valid we can get [depending upon whether we expand $\ln Z_{gr}(\lambda_n, \lambda_p)$ in terms of $\ln Z_{gr}(\lambda'_n, \lambda'_p)$ or vice versa]. $\exp\{[\ln Z_{gr}(\lambda_n, \lambda_p) - \ln Z_{gr}(\lambda'_n, \lambda'_p)]\} = \exp[-\Delta\lambda_n(N-k-1) - \Delta\lambda_p(Z-k)]$, or $\exp[-\Delta\lambda_n(N-k) - \Delta\lambda_p(Z-k-1)]$.

Equation (2.9) can be reduced to $\langle n_{k+1,k} \rangle / \langle n_{k,k+1} \rangle \approx e^{\beta\Delta E_B} e^{\lambda_n - \lambda_p} \approx e^{\beta\Delta E_B} e^{\lambda'_n - \lambda'_p}$.

We will use

$$\frac{\langle n_{k+1,k} \rangle}{\langle n_{k,k+1} \rangle} \approx e^{\beta\Delta E_B} \exp[(\lambda_n + \lambda'_n - \lambda_p - \lambda'_p)/2]. \quad (2.10)$$

Equation (2.10) looks just like a grand canonical result but with an important difference. In the usual grand canonical model λ_n, λ_p would be calculated just once, from Eqs. (2.7) and (2.8) where N and Z are the neutron and proton numbers of the disintegrating system. By contrast, λ_n, λ_p , etc., of Eq. (2.10) are calculated from Eqs. (2.7) and (2.8) for each k and the left-hand sides of Eqs. (2.7) and (2.8) are given by $N-k-1$ and $Z-k$, respectively. The quantity $\lambda_n - \lambda_p$ etc. increases with k with the result that if we try to interpret the canonical results within a usual grand canonical framework one ends up with a larger β , that is, a lower T . Let us expand on this. In the canonical model, the slope of the log of the mirror-yield ratio depends not solely on $\beta\Delta E_B$, but also on another factor that depends on the mass of the fragment, and hence it also varies with ΔE_B . Thus the slope of the curve in the canonical approach does not give a simple measure of T as it would in the grand canonical approach. This is demonstrated in Fig. 2 where it is shown that although the temperature used for the canonical calculation (hence the true temperature) is 5 MeV, deducing the temperature from the slope of the mirror isotope yield ratios (as one would do in a grand canonical formalism) one would arrive at a significantly lower temperature. The best fit (solid line) to the calculated values from the canonical model (solid points) yield a temperature of 3.4 MeV. In the figure we also show that the approximation of Eq. (2.10) as shown by the star points, works quite well.

The dependence of $\lambda_n - \lambda_p$ on k and N, Z where $2k+1$ is the mass number of the emitted particle and N, Z gives the size of the emitting system can be pinned down further. Let Λ_N, Λ_P be the lambda values of the system N, Z . We will write $\lambda_n = \Lambda_N + d\lambda_N$ and $\lambda_p = \Lambda_P + d\lambda_P$. We then have $N-k-1 = \sum i \omega_{i,j} \exp(i\lambda_n + j\lambda_p)$. Expressing λ_n, λ_p in terms of Λ_N, Λ_P and approximating $\exp(d\lambda_P) \approx (1+d\lambda_P)$, etc. we get $-k-1 = A d\Lambda_N + B d\Lambda_P$ where A and B are constants: $A = \sum i^2 \omega_{i,j} \exp(i\Lambda_N + j\Lambda_P)$ and $B = \sum ij \omega_{i,j} \exp(i\Lambda_N + j\Lambda_P)$. Similarly starting from $N-k = \sum j \omega_{i,j} \exp(i\lambda_n + j\lambda_p)$ and expanding as above we get $-k = B d\Lambda_N + C d\Lambda_P$ where $C = \sum j^2 \omega_{i,j} \exp(i\Lambda_N + j\Lambda_P)$. One can now express $d\Lambda_N, d\Lambda_P$ in terms of the constants A, B and C . We get

$$\lambda_n - \lambda_p = \Lambda_N - \Lambda_P + \frac{C-A}{B^2-AC} k + \frac{C+B}{B^2-AC}. \quad (2.11)$$

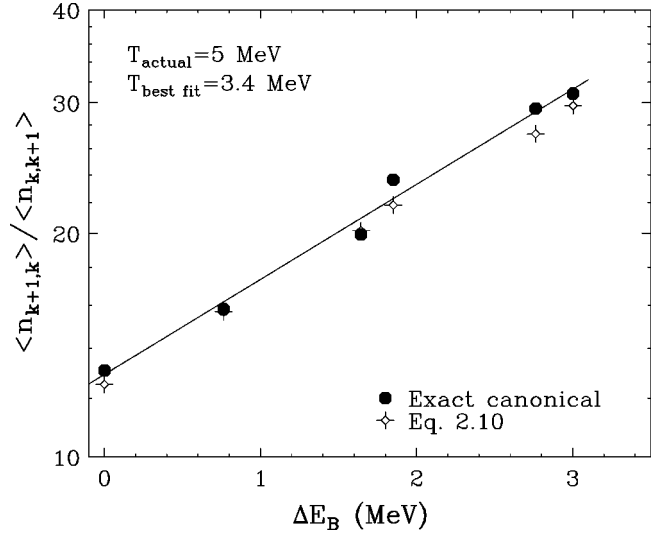


FIG. 2. Exact canonical model calculations for a system of neutron number $N=104$ and $Z=70$ using a freeze-out density of one-quarter of normal density. This simulates central $^{124}\text{Sn} + ^{124}\text{Sn}$ collisions. Lower values of N and Z used in this calculation reflect effects of preequilibrium emissions. The ratios of yields of mirror nuclei are plotted for $a=1, 3, 7, 9, 11$, and 13 . The results for $3, 7$, and 11 can be compared with the experimental results (Fig. 1). $T_{\text{actual}}=5$ MeV is the temperature used in the canonical calculation; $T_{\text{best fit}}$ would be the temperature deduced if one fit the solid points obtained from the canonical calculations, using the grand canonical formula, Eq. (1.4). The results from a saddle-point Eq. (2.10) approximation are also shown.

We can do a similar analysis for $\lambda'_n - \lambda'_p$. Finally we get [compare Eq. (2.10)]

$$(\lambda_n + \lambda'_n - \lambda_p - \lambda'_p)/2 = \Lambda_N - \Lambda_P + \frac{1}{2} \frac{C-A}{B^2-AC} (2k+1). \quad (2.12)$$

Equation (2.12) says that the correction grows like $2k+1 = a$, the mass of the composite. The correction would diminish as the disintegrating system (N, Z) grows. The constants A, B , and C are positive definite and each will become larger and larger as the disintegrating system becomes larger. The correction would disappear in the thermodynamic limit. The actual values of the constants A, B , and C for a finite system depend on many factors: the symmetry energy, the Coulomb energies and N, Z of the disintegrating system.

III. THE ALBERGO TEMPERATURE

The Albergo formula [4] has often been used to extract a temperature from experimental data. The formula is exact if the following two assumptions are valid: (1) the populations of various states are given by the grand canonical model and (2) the secondary decays that will alter these primary populations can be neglected. Define a ratio R

$$R = \frac{Y(A_i, Z_i)/Y(A_i+1, Z_i)}{Y(A_j, Z_j)/Y(A_j+1, Z_j)}, \quad (3.1)$$

where the Y 's are the yields in the ground state. Then, the temperature is given by

$$T = \frac{B}{\ln(sR)}, \quad (3.2)$$

where B is related to binding energies and s to the ground state spins:

$$B = BE(A_i, Z_i) - BE(A_i + 1, Z_i) - BE(A_j, Z_j) + BE(A_j + 1, Z_j), \quad (3.3)$$

$$s = \frac{[2S(A_j, Z_j) + 1]/[2S(A_j + 1, Z_j) + 1]}{[2S(A_i, Z_i) + 1]/[2S(A_i + 1, Z_i) + 1]}. \quad (3.4)$$

Even if the grand canonical model is exact, the change of populations due to secondary decays can cause Eq. (3.2) to give significantly different temperatures from the true grand canonical temperature. This was studied in detail in [15]. It was shown that for large values of B [Eq. (3.3)], the difference between apparent temperature and the true grand canonical temperature decreases. This suggests that while using the Albergo formula to deduce a temperature from experimental data, it is advisable to use pairs that will lead to a large value of B .

Our objective here is different and is complementary to the study made in [15]. The canonical model is obviously more rigorous than the grand canonical model. However, if canonical values for R are used, Eq. (3.2) is no longer strictly correct. Using the primary yields of the ground states, we explore the differences between the deduced temperatures from Eq. (3.2) compared to the actual temperature used in a canonical model. This is shown in Fig. 3, we find that the errors in temperature decrease with increasing B . The inset in Fig. 3 shows the deviation of the canonical Albergo temperature for B greater than 10 MeV. Most of the predicted temperatures are slightly lower than the actual temperature of 5 MeV. The deviations arise from the differences between isotope yields predicted by the canonical and grand canonical models. Not surprisingly, the conclusions of [15] can be applied here.

Extraction of temperature from mirror-yield curves can also be cast in Albergo-type formula. The quantity that corresponds to B of Eq. (3.3) is small for any pair of ratios shown in Fig. 1. The low temperature (3.4 MeV instead of 5 MeV) obtained from the slope of the yield ratios of the mir-

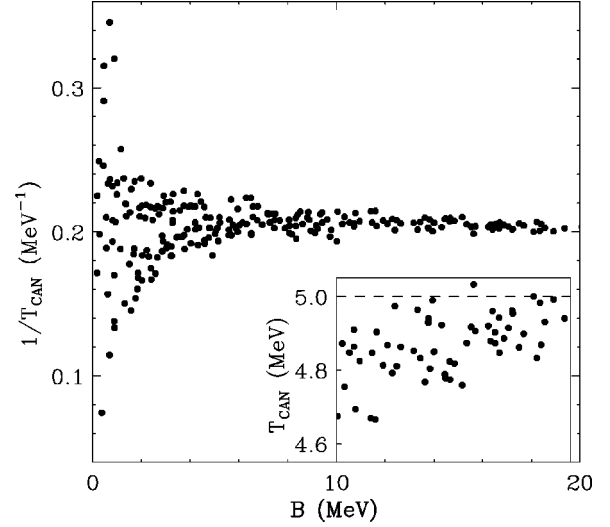


FIG. 3. The inverse Albergo temperature, Eq. (3.2), from the canonical model is plotted as a function of the binding energy difference B . The inset shows the predicted temperature in an expanded scale. The dash line at $T = 5$ MeV is the input temperature to the calculation.

ror nuclei is consistent with the large fluctuations observed in Albergo temperatures extracted from isotopes with small binding energy differences.

IV. THE SCALING LAW

The last quantity we want to investigate is a ratio of two ratios: $(\langle n_{l+m,k} \rangle_2 / \langle n_{l+m,k} \rangle_1) / (\langle n_{l,k} \rangle_2 / \langle n_{l,k} \rangle_1)$ and see if this falls on an exponential as in the grand canonical ensemble. Here the subscripts 1 and 2 refer to two systems: (for example: 2 refers to central collisions of $^{124}\text{Sn} + ^{124}\text{Sn}$ and 1 to central collisions of $^{112}\text{Sn} + ^{112}\text{Sn}$ at 50 MeV/nucleon energy). As this involves two ratios and two different systems, the analysis is considerably more complicated than what we considered before. The ratio R we are after is given by

$$R = \frac{\omega_{l+m,k}^{(2)} \omega_{l,k}^{(1)}}{\omega_{l,k}^{(2)} \omega_{l+m,k}^{(1)}} \frac{Q_{N2-l-m, Z2-k}}{Q_{N2-l, Z2-k}} \frac{Q_{N1-l, Z1-k}}{Q_{N1-l-m, Z1-k}}. \quad (4.1)$$

For central collisions at the same beam energy per particle, the ω factors will give unity. Employing the saddle-point approximation and setting the ratios of the det's as unity as before, we can consider

$$\frac{Q_{N2-l-m, Z2-k}}{Q_{N2-l, Z2-k}} = \frac{\exp\{-[\lambda_n(N2-l-m) + \lambda_p(Z2-k)] + \ln Z_{gr}(\lambda_n, \lambda_p)\}}{\exp\{-[\lambda'_n(N2-l) + \lambda'_p(Z2-k)] + \ln Z_{gr}(\lambda'_n, \lambda'_p)\}}. \quad (4.2)$$

Similarly,

$$\frac{Q_{N1-l-m, Z1-k}}{Q_{N1-l, Z1-k}} = \frac{\exp\{-[\tilde{\lambda}_n(N1-l-m) + \tilde{\lambda}_p(Z1-k)] + \ln Z_{gr}(\tilde{\lambda}_n, \tilde{\lambda}_p)\}}{\exp\{-[\tilde{\lambda}'_n(N1-l) + \tilde{\lambda}'_p(Z1-k)] + \ln Z_{gr}(\tilde{\lambda}'_n, \tilde{\lambda}'_p)\}}. \quad (4.3)$$

We can now indicate how the grand canonical results are recovered. We set $\lambda_n = \lambda'_n, \tilde{\lambda}_n = \tilde{\lambda}'_n$, and $\lambda_n - \tilde{\lambda}_n = \Delta\lambda_n$ then the ratio achieves the exponential character: $R = \exp(m\Delta\lambda_n)$. Experimentally it is found that the relationship $R = \exp(\alpha m)$ where α is a constant independent of l and k is quite well respected. This is not so obvious from Eqs. (4.1), (4.2), and (4.3). We are therefore required to investigate this near independence of the constant α .

If we write in Eq. (4.2) $\lambda'_n = \lambda_n + \Delta\lambda_n$ and expand $\ln Z_{gp}(\lambda'_n, \lambda'_p)$ in terms of $\ln Z_{gp}(\lambda_n, \lambda_p)$ and keep lowest order corrections, the right-hand side of Eq. (4.2) is simply $e^{m\lambda'_n}$. In a similar fashion, the right-hand side of Eq. (4.3) is $e^{m\tilde{\lambda}'_n}$, so that the ratio R of Eq. (4.1) is $\exp[m(\lambda'_n - \tilde{\lambda}'_n)]$ where, of course, the values of $\lambda'_n, \tilde{\lambda}'_n$ are chosen to give neutron numbers $N2-l$ and $N1-l$ and proton numbers $Z2-k$ and $Z1-k$, respectively. Our next task is to verify that $\lambda'_n - \tilde{\lambda}'_n$ is approximately independent of l and k .

We have four equations:

$$\sum i e^{i\lambda'_n + j\lambda'_p} \omega_{i,j} = N2 - l, \quad (4.4)$$

$$\sum j e^{i\lambda'_n + j\lambda'_p} \omega_{i,j} = Z2 - k, \quad (4.5)$$

$$\sum i e^{i\tilde{\lambda}'_n + j\tilde{\lambda}'_p} \omega_{i,j} = N1 - l, \quad (4.6)$$

$$\sum j e^{i\tilde{\lambda}'_n + j\tilde{\lambda}'_p} \omega_{i,j} = Z1 - k. \quad (4.7)$$

Let $\lambda'_n = \tilde{\lambda}'_n + \delta\lambda_n$ and $\lambda'_p = \tilde{\lambda}'_p + \delta\lambda_p$. From Eqs. (4.5) and (4.7), retaining terms to lowest order in $\delta\lambda_p$ and $\delta\lambda_n$ we obtain

$$\delta\lambda_p \sum j^2 e^{i\tilde{\lambda}'_n + j\tilde{\lambda}'_p} \omega_{i,j} + \delta\lambda_n \sum ij e^{i\tilde{\lambda}'_n + j\tilde{\lambda}'_p} \omega_{i,j} = Z2 - Z1. \quad (4.8)$$

In a similar fashion from Eqs. (4.4) and (4.6) we can obtain

$$\delta\lambda_n \sum i^2 e^{i\tilde{\lambda}'_n + j\tilde{\lambda}'_p} \omega_{i,j} + \delta\lambda_p \sum ij e^{i\tilde{\lambda}'_n + j\tilde{\lambda}'_p} \omega_{i,j} = N2 - N1. \quad (4.9)$$

Equations (4.8) and (4.9) can be solved for $\delta\lambda_n$ and $\delta\lambda_p$ and in the lowest order these value are independent of l and k but depend upon $N2, Z2, N1$, and $Z1$. To this order R of Eq. (4.1) is independent of l and k as it is in the usual grand canonical ensemble. This is seen to be obeyed in experiments to a good approximation.

Instead of Eqs. (4.4)–(4.9), one may also consider the following approximation trying to relate to the grand canonical ensemble. Recall that λ'_n, λ'_p are the fugacities of a system which has $N2-l$ neutrons and $Z2-k$ protons. If we denote the fugacities of the system which has $N2$ neutrons and $Z2$ protons by $\Lambda_{N2}, \Lambda_{Z2}$ and employ the same approximate methods used in the discussion leading to Eq. (2.11), we get $\lambda'_n = \Lambda_{N2} + (lC_2 - kB_2)/(B_2^2 - A_2C_2)$. Similarly $\tilde{\lambda}'_n, \tilde{\lambda}'_p$ refers to a system that has $N1-l$ neutrons and $Z1-k$ protons. In an obvious notation we also get $\tilde{\lambda}'_n = \Lambda_{N1} + (lC_1 - kB_1)/(B_1^2 - A_1C_1)$. The quantity of interest is

$$\lambda'_n - \tilde{\lambda}'_n = \Lambda_{N2} - \Lambda_{N1} + \frac{lC_2 - kB_2}{B_2^2 - A_2C_2} - \frac{lC_1 - kB_1}{B_1^2 - A_1C_1}. \quad (4.10)$$

Because of cancellations in the above equation, results again approximate the grand canonical result quite closely.

V. SUMMARY

In summary, we have explored several experimental observables that are sensitive to the isospin effects in multifragmentation. We find that the mirror ratios, isoscaling, and temperatures calculated in canonical model behave similarly as those predicted with the grand canonical model with one significant difference: the temperature deduced from the calculated observables with the canonical model using the rules based on the grand canonical model can be significantly different from the true temperatures.

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- [1] H. Xu *et al.*, Phys. Rev. Lett. **85**, 716 (2000).
[2] M. B. Tsang, W. A. Friedman, C. K. Gelbke, W. G. Lynch, G. Verde, and H. Xu, Phys. Rev. Lett. **86**, 5023 (2001).
[3] J. Randrup and S. E. Koonin, Nucl. Phys. **A356**, 223 (1981).
[4] S. Albergo *et al.*, Nuovo Cimento Soc. Ital. Fis., A **89A**, 1 (1985).
[5] D. R. Bowman *et al.*, Phys. Rev. Lett. **67**, 12 527 (1991).
[6] C. Williams *et al.*, Phys. Rev. C **55**, R2132 (1997).
[7] J. P. Bondorf, A. S. Botvina, A. S. Iljinov, I. N. Mishustin, and K. Sneppen, Phys. Rep. **257**, 133 (1995).
[8] D. H. E. Gross, Phys. Rep. **279**, 119 (1997).
[9] H. S. Xu (private communication).
[10] G. J. Kunde *et al.*, Phys. Lett. B **416**, 56 (1998).
[11] S. Das Gupta and A. Z. Mekjian, Phys. Rev. C **57**, 1361 (1998).
[12] A. Majumder and S. Das Gupta, Phys. Rev. C **61**, 034603 (2000).
[13] P. Bhattacharyya, S. Das Gupta, and A. Z. Mekjian, Phys. Rev. C **60**, 054616 (1999).
[14] M. B. Tsang, C. K. Gelbke, X. D. Liu, W. G. Lynch, W. P. Tan, G. Verde, H. S. Xu, W. A. Friedman, R. Donangelo, S. R. Souza, C. B. Das, S. Das Gupta, and D. Zhabinsky, nucl-ex/0105020.
[15] M. B. Tsang, W. G. Lynch, H. Xi, and W. A. Friedman, Phys. Rev. Lett. **78**, 3836 (1997).